Math 115 Sample Midterm Exam

The midterm exam will consist of 25 *Multiple-Choice* problems. The practice problems below are intended to be representative of what might appear on the exam.

Multiple-Choice Problems

1. The domain of function $f(x) = \dfrac{x+3}{2x^2 - x - 3}$ is

$\quad$ (A) $(-\infty, +\infty)$ $\quad$ (B) $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, +\infty)$ $\quad$ (C) $(-\infty, -1) \cup (-1, +\infty)$

$\quad$ (D) $(-\infty, -1) \cup (-1, \frac{3}{2}) \cup (\frac{3}{2}, +\infty)$ $\quad$ (E) None of the above

2. The domain of function $f(x) = \dfrac{2x}{\sqrt{x^2 - 4}}$ is

$\quad$ (A) $(2, +\infty)$ $\quad$ (B) $(-\infty, -2)$ $\quad$ (C) $(-\infty, -2] \cup [2, +\infty)$

$\quad$ (D) $(-\infty, -2) \cup (2, +\infty)$ $\quad$ (E) None of the above

3. Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = 3x + 5$. Then, $(g \circ f)(x)$ is

$\quad$ (A) $\dfrac{1}{(3x+5)^2}$ $\quad$ (B) $\dfrac{3}{x^2} + 5$ $\quad$ (C) $\dfrac{1}{x^2}$ $\quad$ (D) $3x + 5$

$\quad$ (E) None of the above

4. Find $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

$\quad$ (A) 1 $\quad$ (B) does not exist $\quad$ (C) 6 $\quad$ (D) 5 $\quad$ (E) None of the above

5. Find $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{x}$.

$\quad$ (A) $\dfrac{1}{2}$ $\quad$ (B) limit does not exist $\quad$ (C) 0 $\quad$ (D) -1 $\quad$ (E) None of the above

6. Find $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x + 4}{2x^2 - 3x + 1}$.

$\quad$ (A) 4 $\quad$ (B) $\dfrac{3}{2}$ $\quad$ (C) limit does not exist $\quad$ (D) 0 $\quad$ (E) None of the above

7. Find $\lim\limits_{x \to -\infty} \dfrac{x^2 + 3}{x + 1}$.   (A) 1   (B) 0   (C) 2   (D) 3   (E) None of the above

8. Let $f(x) = \begin{cases} \sqrt{3x + 7} & \text{for } x < 3, \\ 10 - 2x & \text{for } x \geq 3. \end{cases}$

        (A) f is not defined at $x = 3$.   (B) $\lim\limits_{x \to 3} f(x)$ does not exist.

        (C) $f(x)$ is continuous at $x = 3$.   (D) $f(x)$ is differentiable at $x = 3$.

        (E) None of the above

9. Let $f(x) = \dfrac{1}{x}$. When simplified, the difference quotient $\dfrac{f(x + h) - f(x)}{h}$ becomes

        (A) 1   (B) $\dfrac{-1}{x(x + h)}$ (C) $\dfrac{1}{h^2}$ (D) 0   (E) None of the above

10. What is the marginal profit in terms of the cost $C(x)$ to produce x units and the unit price $p(x)$ at which x units will sell ?

        (A) $p'(x) - C'(x)$ (B) $p'(x) + xp(x) - C(x)$ (C) $x(p(x) - C(x))$

        (D) $p(x) + xp'(x) - C'(x)$ (E) None of the above

11. Given the demand equation $4x + 2p - 36 = 0$ and the supply equation $2x - p + 10 = 0$, where p is the unit price and x represents the quantity, find the equilibrium quantity and the equilibrium price.

        (A) $x = 2$ and $p = 14$ (B) $x = 3$ and $p = 16$ (C) $x = 4$ and $p = 18$

        (D) $x = 18$ and $p = 4$ (E) None of the above

12. It is known that $\lim\limits_{x \to 2^+} f(x) = 3$, $\lim\limits_{x \to 2^-} f(x) = 3$, and $f(2) = 1$. Which of the following statements is False ?

        (A) $f(x)$ is discontinuous at $x = 2$.   (B) The graph of $f(x)$ is broken at $x = 2$.

        (C) $\lim\limits_{x \to 2} f(x)$ exists.   (D) $f(x)$ is differentiable at $x = 2$.   (E) $f(x)$ is defined at $x = 2$.

13. A major corporation is building a complex of homes, offices, stores and schools in a rural community. As a result of this development, the planners have estimated that the population (in thousands), t years from now will be given by

$$P(t) = \frac{20t^2 + 100t + 250}{t^2 + 5t + 50}.$$

What will be the population in the long run?

$$\text{(A) } 3000 \quad \text{(B) } 20000 \quad \text{(C) } 5000 \quad \text{(D) } 60000$$

$$\text{(E) None of the above}$$

14. For $f(x) = \sqrt{2 + \sqrt{x}}$, evaluate $f'(4)$.

$$\text{(A) } \frac{1}{64} \quad \text{(B) } \frac{1}{16} \quad \text{(C) } \frac{1}{4} \quad \text{(D) } \frac{1}{2} \quad \text{(E) None of the above}$$

15. Let $f(x) = |x|$. Then,

$$\text{(A) } f \text{ is not defined at } x = 0 \quad \text{(B) } f \text{ has no limit at } x = 0$$

$$\text{(C) } f \text{ is not continuous at } x = 0 \quad \text{(D) } f \text{ has no derivative at } x = 0$$

$$\text{(E) None of the above}$$

16. An equation for the line tangent to the curve $y = (x^2 + x + 1)(x^3 - 2x + 2)$ at the point $(1, 3)$ is

$$\text{(A) } y = 2x + 1 \quad \text{(B) } y = 3x \quad \text{(C) } y = 6x - 3 \quad \text{(D) } y = 7x - 4$$

$$\text{(E) None of the above}$$

17. Suppose that $F(x) = f(x^2 + 1)$ and $f'(2) = 3$. Find $F'(1)$.

$$\text{(A) } 3 \quad \text{(B) } 4 \quad \text{(C) } 5 \quad \text{(D) } 6 \quad \text{(E) None of the above}$$

18. Suppose $h = f \circ g$. Find $h'(0)$ given that $f'(5) = -2$, $g(0) = 5$ and $g'(0) = 3$.

$$\text{(A) } -6 \quad \text{(B) } 18 \quad \text{(C) } -10 \quad \text{(D) } 30 \quad \text{(E) None of the above}$$

19. Find $\dfrac{dy}{dx}$ in terms of x and y when x and y are related by the equation $x^2y - y^3 = 2$.

$$\text{(A) } \frac{xy}{3y^2 + x^2} \quad \text{(B) } \frac{x}{x - 3y} \quad \text{(C) } \frac{2xy}{3y^2 - x^2} \quad \text{(D) } \frac{x^2 + y^2}{2x + y}$$

$$\text{(E) None of the above}$$

20. Two ships leave the same port at noon. Ship A sails north at 15mph, and ship B sails east at 12mph. How fast is the distance between them changing at 1p.m?

(A) 30.5   (B) 19.2   (C) 14.4   (D) 15.3

(E) None of the above

21. The third derivative of $f(x) = \frac{1}{x}$ is

(A) $\frac{-1}{x^2}$ (B) $\frac{-2}{x^3}$ (C) $\frac{2}{x^3}$ (D) $\frac{1}{x^2}$ (E) None of the above

22. The distance s (in feet) covered by a car t seconds after starting from rest is given by $s = -t^3 + 8t^2 + 20t$. Find the car's acceleration at time t.

(A) $-t^3 + 8t^2 + 20t$ (B) $-3t^2 + 16t + 20$ (C) $-6t + 16$ (D) $-6t + 36$

(E) None of the above

23. Find an equation of the tangent line at the point $(1, 32)$ of the graph of $y = x(x+1)^5$.

(A) $y - 32 = (x+1)^4(6x+1)$ (B) $y + 32 = (x+1)^4(6x+1)$ (C) $y = 112x - 80$

(D) $y = 112x + 80$ (E) None of the above

24. The owner of a vineyard estimates that if 10,000 bottles of wine were produced this season, then the profit would be $ 5/bottle. But if more than 10,000 bottles were produced, then the profit/bottle would drop by $ 0.0002 for each additional bottle sold. What is the rpofit he can expect from the sale of 16,000 bottles of wine from his vineyard?

(A) $60,800$ (B) $50,800$ (C) 68.800 (D) $58,800$

(E) None of the above

25. The unit price p and the quantity x demanded are related by the demand equation $2p(x + 1) = 20$. Find the marginal revenue function.

(A) $\frac{10}{(x+1)^2}$ (B) $\frac{20x + 10}{(x+1)^2}$ (C) $\frac{-10}{(x+1)^2}$ (D) $\frac{10}{x+1}$

(E) None of the above