Math 115 Sample Final Exam

The final exam will consist of 30 *Multiple-Choice* problems. The practice problems below are intended to be representative of what might appear on the actual exam.

Multiple-Choice Problems

1. Let $f(x) = \frac{\sqrt{x+1}}{x-2}$. The domain of f is

   (A) $(-\infty, 2)$ and $(2, +\infty)$ (B) $(-\infty, 2]$ and $[2, +\infty)$ (C) $[-1, 2)$ and $(2, +\infty)$

   (D) $[-1, +\infty)$ (E) None of the above

2. Let $f(x) = \frac{x}{x^2+1}$ and $g(x) = \frac{1}{x}$. Then, $(g \circ f)(x)$ is

   (A) $\dfrac{x}{x^2+1}$ (B) $\dfrac{1}{x}$ (C) $x + \dfrac{1}{x}$ (D) x (E) None of the above

3. Evaluate $\lim_{x \to 5} \frac{x^2-2x-15}{x-5}$.

   (A) 3 (B) 8 (C) 0 (D) Does not exist (E) None of the above

4. Find the horizontal asymptotes of function $f(x) = \dfrac{\sqrt{x^4+1}}{1+4x^2}$.

   (A) $y = 1$ (B) $y = \dfrac{1}{4}$ (C) $x = 1$ (D) No horizontal asymptotes (E) None of the above

5. Find the vertical asymptotes of function $f(x) = \frac{2+x}{(1-x)^2}$.

   (A) $x = -2$ (B) $x = 1$ (C) $y = 0$ (D) No vertical asymptotes (E) None of the above

6. Suppose that $F(x) = f(x)^2 + 1$, f(1) = 1, and $f'(1) = 3$. Find $F'(1)$.

   (A) 3 (B) 4 (C) 5 (D) 6 (E) None of the above

7. The unit price p and the quantity x demanded are related by the demand equation $50 - p(x^2 + 1) = 0$. Find the revenue function $R = R(x)$.

   (A) $\dfrac{50x}{x^2+1}$ (B) $\dfrac{50}{x^2+1}$ (C) $\dfrac{x}{x^2+1}$ (D) $\dfrac{x^2+1}{50}$ (E) None of the above

8. Find the marginal revenue for the revenue function found in Problem 7.

(A) $\dfrac{-100x}{(x^2+1)^2}$ (B) $\dfrac{1-x^2}{(x^2+1)^2}$ (C) $\dfrac{x}{25}$ (D) $\dfrac{50(1-x^2)}{(x^2+1)^2}$ (E) None of the above

9. The line tangent to $y = x^2 - 3x$ through the point (1, -2) has the equation

(A) $y = x - 3$ (B) $y + 2 = (2x - 3)(x - 1)$ (C) $y = -x - 1$

(D) $y - 2 = (2x - 3)(x - 1)$ (E) None of the above

10. Find an equation of the tangent line to the graph of $y = \ln(x^2)$ at the point $(2, \ln 4)$.

(A) $y = x + 2 - \ln 4$ (B) $y = \dfrac{2}{x}(x - 2) - \ln 4$ (C) $y = \dfrac{2}{x}(x - 2) + \ln 4$

(D) $y = x - 2 + \ln 4$ (E) None of the above

11. Find $\frac{dy}{dx}$ in terms of x and y when x and y are related by the equation $x^{1/3} + y^{1/3} = 1$.

(A) $-\left(\dfrac{x}{y}\right)^{2/3}$ (B) $-\left(\dfrac{y}{x}\right)^{2/3}$ (C) $-\left(\dfrac{x}{y}\right)^{1/3}$ (D) $-\left(\dfrac{y}{x}\right)^{1/3}$

(E) None of the above

12. A 10-foot long ladder is leaning against a wall. If the bottom of the ladder is pulled away from the wall at a rate of 3 feet per second, how fast (in feet per second) is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

(A) 7/4 (B) 3/2 (C) 9/4 (D) 3/8 (E) 7/2

13. The absolute extrema of the function $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}$ on [0, 3] are

(A) absolute min. value: $-\dfrac{3}{2}$; absolute max. value: $\dfrac{9}{2} - 2\sqrt{3}$

(B) absolute min. value: 0; absolute max. value: 3

(C) absolute min. value: 0; no absolute max. value

(D) no absolute min. value ; absolute max. value: 3

(E) None of the above

14. Find the absolute extrema of the function $f(t) = te^{-t}$.

(A) absolute min. value: 0; absolute max. value: $\dfrac{1}{e}$

(B) absolute min. value: 0; no absolute max. value

(C) no absolute min. value ; absolute max. value: $\dfrac{1}{e}$

(D) no absolute min. value ; no absolute max. value

(E) None of the above

15. Let $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, 1)$ and on $(1, \infty)$

(B) increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$

(C) decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$

(D) decreasing on $(-\infty, 1)$ and on $(1, \infty)$

(E) None of the above

16. Let the function f be defined in Problem 15. Find the intervals where f is concave upward and where it is concave downward.

(A) concave upward on $(-\infty, 1)$ and on $(1, \infty)$

(B) concave upward on $(-\infty, 1)$ and downward on $(1, \infty)$

(C) concave downward on $(-\infty, 1)$ and upward on $(1, \infty)$

(D) Concave downward on $(-\infty, 1)$ and on $(1, \infty)$

(E) None of the above

17. Let the function f be defined in Problem 15. Find the inflection points, if any.

(A) $(x, y) = (1, f(1))$ (B) $(x, y) = (\dfrac{1}{2}, f(\dfrac{1}{2}))$ (C) $(x, y) = (0, f(0))$

(D) No inflection points (E) None of the above

18. Let $f(x) = x \ln x$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, \frac{1}{e})$ and decreasing on $(\frac{1}{e}, \infty)$

(B) decreasing on $(-\infty, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$

(C) increasing on $(0, \frac{1}{e})$ and decreasing on $(\frac{1}{e}, \infty)$

(D) decreasing on $(0, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$

(E) None of the above

19. Suppose that f is defined in Problem 18. Determine the intervals of concavity for the function.

(A) concave upward on $(0, \infty)$

(B) concave downward on $(0, \infty)$

(C) concave upward on $(0, \frac{1}{e})$; concave downward on $(\frac{1}{e}, \infty)$

(D) concave downward on $(0, \frac{1}{e})$; concave upward on $(\frac{1}{e}, \infty)$

(E) None of the above

20. Suppose that f is defined in Problem 18. Find the inflection points, if any.

(A) $(x, y) = (\frac{1}{e}, f(\frac{1}{e}))$ (B) $(x, y) = (1, f(1))$ (C) $(x, y) = (e, f(e))$

(D) No inflection points (E) None of the above

21. Find the derivative of function $y = 10^x$. (Hint: use logarithmic differentiation.)

(A) $y' = 10^x \ln 10$ (B) $y' = 10^x$ (C) $y' = 10^x \ln e$

(D) the derivative does not exist (E) None of the above

22. For $f(x) = \dfrac{(x^3 + x + 1)^{1/3}(2x + 3)}{(x^2 + 1)^2}$, calculate $f'(0)$. (Hint: Use logarithmic differentiation.)

(A) 11/3 (B) 10/3 (C) 3 (D) 8/3 (E) 7/3

23. An open box is to be made from a square sheet of tin measuring 12 in.$\times$12 in. by cutting out a square of side x inches from each corner of the sheet and folding up the four resulting flaps. To maximize the volume of the box, x should be

(A) 1 in. (B) 2 in. (C) 3 in. (D) 4 in. (E) None of the above

24. Postal regulations specify that a parcel sent by parcel post may have a combined length and girth of no more than 108 inches. Find the dimensions of the cylindrical package of greatest volume that may be sent through the mail. (In the answers, r is the radius and l is the length.)

(A) $r \times l = \dfrac{35}{\pi} \times 37$ (B) $r \times l = \dfrac{36}{\pi} \times 36$ (C) $r \times l = \dfrac{37}{\pi} \times 35$

(D) $r \times l = \dfrac{38}{\pi} \times 34$ (E) None of the above

25. The velocity of a car (in feet/second) t seconds after starting from rest is given by the function $f(t) = 2\sqrt{t}$ $(0 \le t \le 30)$. Find the car's position at any time t.

(A) $\frac{4}{3}t^{3/2} + C$ (B) $\frac{4}{3}t^{3/2}$ (C) $\frac{4}{3}t^{1/2} + C$ (D) $\frac{4}{3}t^{1/2}$ (E) None of the above

26. Evaluate $\int (\sqrt{x} - 2e^x)\, dx$.

(A) $\frac{2}{3}x^{3/2} - 2e^x$ (B) $\frac{2}{3}x^{3/2} - 2e^x + C$ (C) $\frac{3}{2}x^{2/3} - 2e^x$

(D) $\frac{3}{2}x^{2/3} - 2e^x + C$ (E) None of the above

27. Evaluate $\int xe^{-x^2} dx$.

(A) $-e^{-x^2} + C$ (B) $-\frac{1}{2}e^{-x} + C$ (C) $(1 - 2x^2)e^{-x^2} + C$ (D) $-\frac{1}{2}e^{-x^2} + C$

(E) None of the above

28. Calculate $\int_1^8 \left(4x^{1/3} + \frac{8}{x^2}\right) dx$.

(A) 49 (B) 50 (C) 51 (D) 52 (E) None of the above

29. Evaluate $\int_0^3 |1 - x| dx$.

(A) 3/2 (B) 5/2 (C) 7/2 (D) 9/2 (E) None of the above

30. Find the area of the region under the graph of function $f(x) = x^2$ on the interval $[0, 1]$.

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{5}$ (E) None of the above