

Exam 1, Mathematics 121 Fall 2007

Name:

Show all work for full credit. Please be very neat and clear in your steps.

1.a.(10 points) Use the definition of the derivative to find $f'(4)$ when $f(x) = \frac{1}{x+2}$

b.(3 points) Find an equation of the tangent line at $(4, \frac{1}{6})$

2.(10 points) Consider the function $f(x) = 5x^5 + 3x^4 + 1$. Find an integer n such that $f(n) < 0$ and $f(n+1) > 0$. Show that there is a real number c such that $n < c < n+1$ and $f(c) = 0$ (name the theorem used)

3.(3 points each) State either T (True) or F (False) for the following statements.

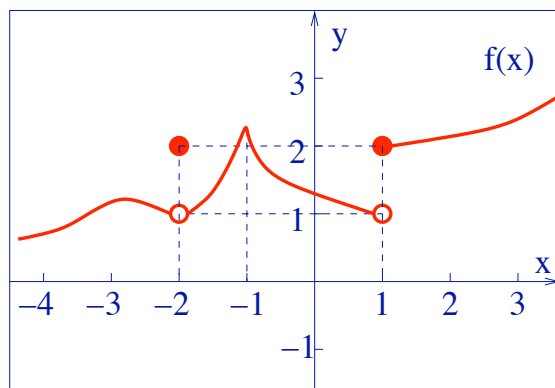
(T)(F) If $\lim_{x \rightarrow 2^+} f(x) = \pi$ and $\lim_{x \rightarrow 2^-} f(x) = 3.1415926$, then $\lim_{x \rightarrow 2} f(x) = \pi$ and f is continuous at $x = 2$.

(T)(F) If $\lim_{x \rightarrow 2^+} f(x) = \pi$ and $\lim_{x \rightarrow 2^-} f(x) = 3.14$, then $\lim_{x \rightarrow 2} f(x)$ does not exist and f is not continuous at $x = 2$.

(T)(F) All polynomial functions are continuous functions.

(T)(F) $f(x) = \frac{1}{x^2 - 1}$ is continuous everywhere except $x = 1$.

4. (15 points) Consider the function f defined by the following graph.



(a) f is not continuous at $x =$ _____.

(b) f does not have a derivative at $x =$ _____.

(c) $\lim_{x \rightarrow -2} f(x) =$ _____.

(d) $\lim_{x \rightarrow 1^-} f(x) =$ _____.

(e) $f(-2) =$ _____.

5.(5 points each) Compute each of the following limits exactly (show your work) or state DNE:

a. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{(x - 2)^2}$

b. $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + x}$

c. $\lim_{x \rightarrow \infty} \frac{2 - x^2}{\pi x^2 - 3x + 1}$

d. $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 1}{x + \sqrt{x}}$

6.(5 points each) Find the interval(s) on which the function is continuous.

a. $f(x) = \frac{\ln x}{\sqrt{1-x}}$

b. $h(t) = \sqrt{16-t^2}$

7.(10 points) Compute $\lim_{x \rightarrow 1} (x-1)^2 \sin \frac{1}{x-1}$ (name the theorem used).

8.(10 points) Let

$$f(x) = \begin{cases} x+1 & \text{when } x \leq 1, \\ 5+ax^2 & \text{when } x > 1. \end{cases}$$

For which value of a will f be continuous at $x = 1$?