DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 121/141, Fall 2008
FINAL EXAM

Your Name: ____________________ **Instructor:** ____________________

Problem	Points	SCORE
1 - 8	80	
9	25	
10	25	
11	25	
12	25	
13	30	
14	30	
15	30	
16	30	
TOTAL	300	

Part A - Multiple Choice Examination

Each right answer is worth 10 points. Select only **one** ans

1. The curve $y = xe^{-2x}$ has a horizontal tangent line when x is

   (a) 1 (b) 0 (c) 2
   (d) 0.5 (e) -1 (f) None of the above

2. $f(x)$ is differentiable over the interval $(0, 3)$ with $f(0) = 0$, $f(2) = 3$, and $f(3) = 3$. Which of the following is NOT necessarily true?

   (a) There is a $0 < c < 3$ with $f(c) = 2.5$.
   (b) $f'(x) = 3$ at some point in $[0, 3]$.
   (c) $f'(x)$ is positive at some point in $(0, 3)$.
   (d) $f'(x) = 0$ at some point in $[0, 3]$.
   (e) All of the above statements must be true.

3. Which integral matches the Riemann sum?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{2n+i}{n}\right)^2$$

(a) $\int_0^1 x\,dx$ (b) $\int_2^3 (2x+1)^2\,dx$ (c) $\int_0^1 \frac{dx}{x^3}$ (d) $\int_0^1 (2+x)^2\,dx$

4. What is the average value of $f(x)$ over the interval $[2,4]$ if $f(2)=2$, $f(4)=8$, and $f'(x)=x$?

(a) $\frac{28}{3}$ (b) 5 (c) $\frac{7}{3}$ (d) $\frac{14}{3}$ (e) $\frac{56}{3}$ (f) none of the above.

5. $\lim_{x\to 1} \frac{|x-1|}{x^2-1} =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) ∞ (d) $-\infty$
(e) does not exist.

6. $\displaystyle\lim_{h\to 0} \frac{1}{h} \ln\left(\frac{3+h}{3}\right) =$

(a) $\frac{1}{3}$ (b) 1 (c) e^3 (d) $\ln 3$ (e) does not exist.

7. If f, f' and f'' are continuous functions on the interval [a,b], such that $f'(a) > 0$, $f'(b) < 0$, and $f''(x) < 0$ for all x in (a,b), then which of the following must be true:

(a) There exist a number c, in (a,b), such that $f(c) = 0$.
(b) There exist a number c in (a,b) such that $f(c) = \frac{a+b}{2}$.
(c) $f(x)$ has a local minimum in the interval (a,b).

(d) $f(x)$ has a local maximum in the interval (a,b).

(e) None of the above.

8. Use Newton's method to solve the equation $x^2 - e^x = 0$, starting with $x_1 = 1$. The next iteration x_2 is:

(a) 1 (b) 0 (c) $\frac{1}{1-e}$ (d) $\frac{1}{2-e}$ (e) None of the above.

9. (**25 points**) Evaluate

$$\int_1^{\sqrt{3}} \arctan(1/x)dx.$$

10. (**25 points**) A rectangular box with a square base and open top is constructed to have volume of 625 cubic inches. The material used to make the bottom costs 4 cents per square inch and the material used to make the sides costs 2 cents per square inch. Find the dimensions of the box that minimizes the total costs. Justify your answer.

11. (**25 points**) A pile of sand with volume $12,000,000\pi$ cubic feet is collapsing. As it collapses it remains in the shape of a cone so that the height decreases as the radius increases and the total volume of the sand remains constant. If the height is decreasing at a constant rate of $4ft/min$, how fast is the radius increasing at the instant the height is 90 feet and the radius is 200 feet?

12. (**25 points**) For the implicitly defined curve

$$x^y = y^x,$$

write the equation of the tangent line at the point $P(2,2)$.

13. (**30 points**) Compute the limit

$$\lim_{x\to 1+}\left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right)$$

14. (**30 points**) A water tank in the shape of an inverted cone is buried such that the top of the tank is located 3 meters below the ground. The tank has a radius of 10 meters and is 15 meters deep. If the tank is filled to a depth of 12 meters, how much work, measured in joules, is required to empty the tank by pumping all of the water to ground level? (Assume density of water is 1000 kg/m^3 and gravitational acceleration is 9.8 m/sec^2.)

15. (**30 points**) Consider the region R in the first quadrant bounded by $y = x^3$ and $y = x$.

- (15 points) Compute the center of mass of the region.
- (15 points) Compute the volume of the solid obtained by rotating the region R about the y-axis.

16. (**30 points**) Evaluate the improper integral or otherwise show it is divergent

$$\int_0^\infty x^2 e^{-x} dx,$$