# COURSE: MATH 851, FALL 2008

## TOPICS IN DYNAMICAL SYSTEMS

INSTRUCTOR: Milena Stanislavova

TIME AND PLACE: T&R 2:30-3:45, 564 Snow

TEXT: P.Hislop and I.Sigal, Introduction to Spectral Theory with applications to Schrödinger operators, Springer Texts in Applied Mathematics 113

PREREQUISITES: Math 220 or 320, 221, 321 (Math 850 and 960 recommended, but not required)

CREDIT HOURS: 3

GRADING: Your grade in this class will be based on several homework assignments and a final project.

The class will cover various aspects of the theory of spectral, linear and nonlinear stability of special solutions to partial differential equations. As a first ingredient, the spectral theory of linear operators will be developed, including point and essential spectrum for self-adjoint, compact and locally compact operators and their perturbations, properties of the resolvent and Riesz projections, Kato-Rellich and Weyl theorems. Information about the spectrum is usually the first step in investigating the stability/instability of complicated nonlinear dynamical systems. The emphasis of the course will be to demonstrate the ingredients in the spectral analysis such as the notions of spectral stability, localization, spectral deformation, perturbations and resonances. These have applications in many areas of mathematical physics, but we will develop the methods so that we can analyze the Schrödinger operators with the use of geometric methods. The discrete and essential parts of the spectrum will be characterized and studied using properties of the resolvent, the Riesz projections, Weyl theorem etc. We will work with general closed operators as well as with the special but important class of self-adjoint operators on a Hillbert space.

We will discuss several notions of stability for the discrete and essential spectrum under various kinds of perturbations. Several important examples will be studied in the second part of the class, including linear stability of pulses in dissipative systems, stability of waves in Hamiltonian equations and the relation between linear and nonlinear stability. This will be done by introducing a variety of contemporary tools such as the Fredholm properties and exponential dichotomies of the operators, the Evans function and several decomposition techniques. If time permits, several recent results in that direction will be studied and presented as final projects.

Math 850 and 851 will satisfy the advanced sequence requirement for the Ph.D. in the Department of Mathematics.

## OUTLINE OF THE MATERIAL

1. Basic properties of the spectrum of a linear operator, decomposition into essential and discrete, resolvents and resolvent estimates.
2. Schrödinger operators with potentials that are larger than a constant at infinity. Exponential decay of eigenfunctions.
3. Operators on Hilbert spaces, self-adjoint operators, Riesz projections and isolated points of the spectrum.
4. Essential spectrum and Weyl's criterion. Self-adjoint, compact and locally compact operators.
5. Semiclassical analysis of the Schrödinger operators.
6. The Kato-Rellich Theorem for relatively bounded operators, Weyl's Theorem for relatively compact operators.
7. Perturbation theory for relatively bounded perturbations.
8. Spectral bounds, growth bounds and spectral mapping theorems.
9. Gearhart-Prüss theorem in stability for wave equations.
10. Linearized stability of travelling waves.
11. Stability analysis of pulses via the Evans function in dissipative systems.
12. Presentation of project papers.