DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 220 - SPRING 2005 - FINAL EXAM

Your Name: _____

On this exam, you may use a calculator and some formula
notes.

It is not sufficient to just write down the answers. You
must explain how you arrived at your answers and how you know they are correct.

$$
\begin{array}{lll}
1 & (35) & \underline{\hspace{2cm}} \\
2 & (35) & \underline{\hspace{2cm}} \\
3 & (35) & \underline{\hspace{2cm}} \\
4 & (35) & \underline{\hspace{2cm}} \\
5 & (35) & \underline{\hspace{2cm}} \\
6 & (35) & \underline{\hspace{2cm}} \\
7 & (35) & \underline{\hspace{2cm}} \\
8 & (35) & \underline{\hspace{2cm}} \\
9 & (35) & \underline{\hspace{2cm}} \\
10 & (35) & \underline{\hspace{2cm}} \\
\text{Total} & (350) & \underline{\hspace{2cm}} \\
\end{array}
$$

- **1. (35 points)** Solve the initial-value problem and draw the graph of the solution

$$ty' + 2y = 4t^2 \quad y(1) = 2.$$

- **2. (35 points)** The population of mosquitoes in a certain area increases at a rate proportional to the current population and, in the absence of other factors, the population triples each week. There are 300 000 mosquitoes in the areal initially, and predators (birds, etc.) eat 50 000 mosquitoes a day. Determine the population of mosquitoes in the area at any time.

- **3. (35 points)** Find the solution of the initial-value problem

$$\left| \begin{array}{l} 2y'' - 3y' + y = 0 \\ y(0) = 2 \\ y'(0) = 1/2 \end{array} \right.$$

- **4. (35 points)** Find the general solutions of

(i) $\quad y'' + 4y = 0;$ $\quad$ and $\quad$ (ii) $\quad y'' + 4y = \cos(2x) + e^x.$

- **5. (35 points)** Solve the initial-value problem

$$\left| \begin{array}{l} t^3 y' + 4t^2 y = e^{-t} \\ y(-1) = 0 \end{array} \right.$$

- **6. (35 points)** Solve the initial-value problem

$$\left| \begin{array}{l} x + ye^{-x}y' = 0 \\ y(0) = 1 \end{array} \right.$$

- **7. (35 points)** Find the general solution of the linear system

$$\begin{cases} x' &= 2x - 3y \\ y' &= 4x - 6y. \end{cases}$$

- **8. (30 points)** Find the solution of the initial-value problem

$$\begin{cases} x_1' &= x_1 - 5x_2 \\ x_2' &= x_1 - 3x_2. \end{cases}$$

  where

$$x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Draw the graph of the solution and describe its behavior for increasing t.

- **9. (35 points)** Let

$$\left|\begin{array}{l} y'' - 4y' + 4y = 0 \\ y(0) = 0 \\ y'(0) = a \end{array}\right.$$

find the only value of the parameter a for which the solution stays bounded as $t \to \infty$.

- **10. (35 points)** Solve the system of equations

$$\begin{cases} x_1' &= x_1 - 4x_2 \\ x_2' &= 4x_1 - 7x_2 \end{cases}$$